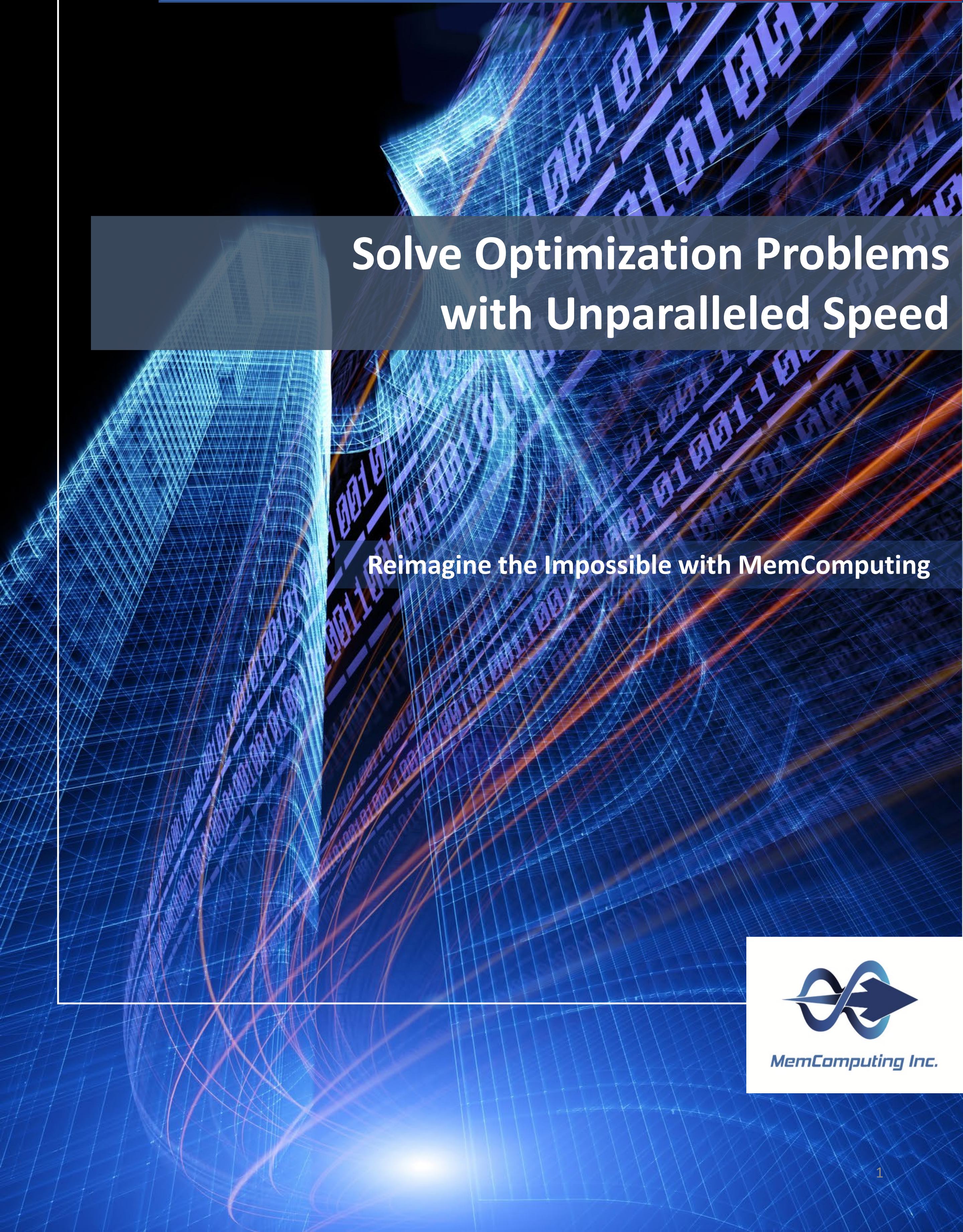
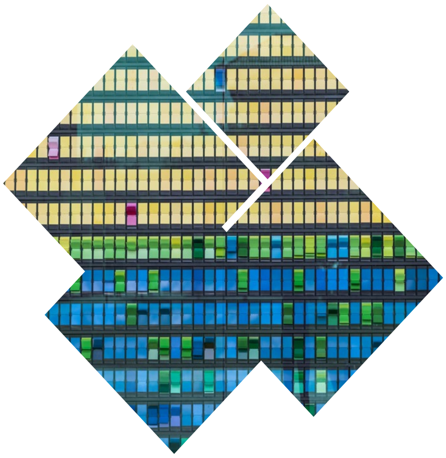
White Paper



Solve Optimization Problems with Unparalleled Speed

Reimagine the Impossible with MemComputing

MemComputing Inc.



EXECUTIVE SUMMARY

MemComputing aims at solving complex optimization problems for a wide-range of applications across commercial industries and scientific communities by orders of magnitudes faster than it is presently possible.

In recent years, the world has witnessed an explosive growth in data, descriptively captured as Big Data. While Big Data has changed the landscape of every commercial industry to varying degrees, it has also created significant issues for optimization. Many optimization problems seek a solution that satisfies as many constraints as possible. However, this task becomes extremely difficult because the time it takes for current technology to solve optimization problems increases exponentially with each additional input. **This means that computation times using traditional optimization strategies easily reach millions of years, even exceeding the age of the universe, which makes them impractical to implement**. Indeed, the need for time-sensitive solutions to complex optimization problems has led to a reliance on heuristics that generate approximations for the optimal solution.

With a physics-inspired approach, MemComputing presents an answer to these difficult problems in the form of a powerful software-based coprocessor that bypasses the problem of impossibly long computation times and provides better results. In fact, the MemComputing MemCPU™ Coprocessor MemCPU Coprocessor performs orders of magnitude faster than the fastest solvers from the 2016 MaxSAT competition and finds better approximations for specific problems when best solvers could not.[1]

Our technology can be realized in hardware with non-quantum electronic components which makes it readily scalable and cost-effective. Furthermore, the proprietary MemComputing circuitry can be efficiently simulated in software. As a matter of fact, MemCPU Coprocessor is currently available both as Software as a Service (SaaS) model and through custom Software Development Kits (SDKs).

Here at MemComputing, we are not simply introducing lightning-fast computational solutions; **our technology holds immense disruptive potential for optimization problems with a fundamental paradigm shift in the ways in which computation is performed**.

1 The Problem

Optimization is a core component of every data-based scientific discipline and industry. The **difficulty of computation is especially prominent in a set of problems classified as NP-hard**. Currently, no algorithms exist for NP-hard problems that find a solution in polynomial time. In other words, there is no known way to find the optimal solution that meets the maximum number of constraints within a reasonable amount of time. Furthermore, for many cases, even an approximation to the optimal solution is challenging to discover. In fact, for the hardest cases, as the number of inputs and constraints increase linearly, the computational time to find even an approximation to the optimal solution grows exponentially to the point where computations would take millions of years, or even longer than the age of the Universe, to complete.

To avoid these long computations, **research has focused on developing sophisticated heuristics that approximate reasonably good solutions in a feasible amount of time**. For commercial industries such as transportation, routing, and software or hardware upgrades (to name a few), better approximations at a fraction of the current computation time translates to hundreds of millions of dollars saved annually and improved operational performance.

There are many groups working on solving this high-value challenge. However, each present shortcomings in their approach:

- **Heuristics algorithms research and development** have historically bore the responsibility for solving complex optimization problems. While the implementation of heuristic algorithms is the most widely accepted solution across academia and commercial industries, the problems of heuristics (namely exponentially increasing computation time and challenges in refining candidate solutions) limit their performance.

- **Cloud computing** allows users to tap into a shared system resources provided via the internet. This allows organizations to outsource the burden of maintaining computer infrastructure which frees up resources to be dedicated to scaling. Even though cloud computing can supplement an organization’s processing power, it does not resolve the need for heuristic algorithms and pose significant privacy concerns as the cloud vendor enjoy sole access to backend infrastructure.

- **Quantum computing** theoretically presents the ability to store and process huge amounts of information through quantum bits that can exist in a superimposition of 0s and 1s. However, there are currently significant physics, engineering, and scaling challenges preventing the realization of fully functional scalable quantum computers.

2 The MemComputing Performance Advantage

The unprecedented performance of the MemComputing technology is based on the pioneering theoretical concept of **universal memcomputing machines** (UMMs). UMMs are a class of scalable memcomputing machines built with interconnected memory units (memprocessors) capable of performing computation.

UMMs are practically realized in the form of **digital memcomputing machines** (DMMs). DMMs harness the power of **self-organizing logic circuits** (SOLCs) which are differentiated from traditional circuits through the unique properties of **self-organizing logic gates** (SOLGs) used in their construction. These SOLGs, in turn, can be realized in hardware with available technology, or simulated efficiently in software.

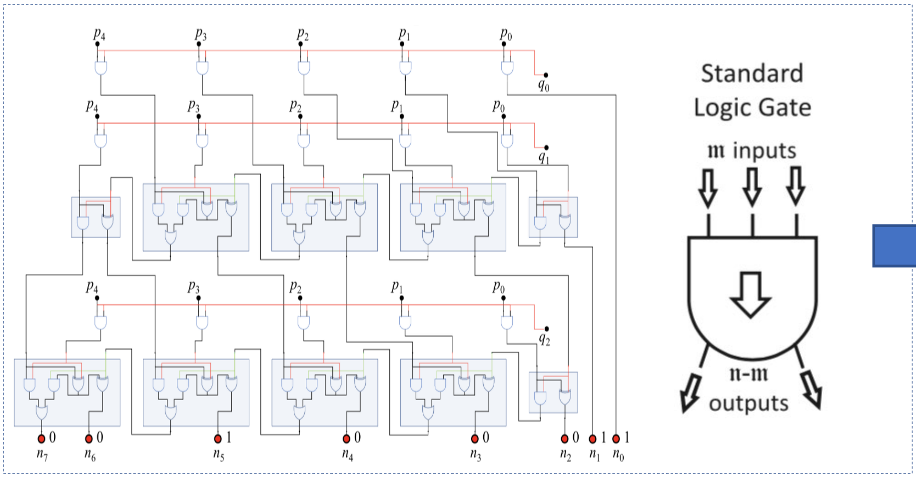
The proprietary components of the MemComputing system allows for a non-combinatorial approach for optimization problems which removes the dependency on heuristic algorithms and stochastic local searches. The features of SOLGs, the foundational component of MemComputing capabilities, provide the ultimate computational technology that is cost-effective, reduces computation time by orders of magnitude, and increases the quality of the solution to complex optimization problems.

Self-Organizing Logic Gates

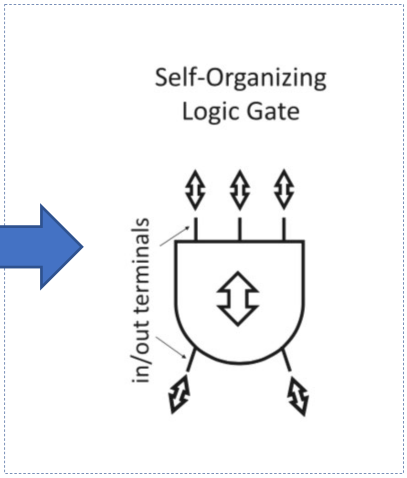
Logic gates are the building blocks for digital circuits.
Traditional logic gates are essentially miniature electric circuits that receive incoming electrical currents (inputs) and send an outgoing electrical current (output) based on what came in. The role of logic gates is to perform a logical operation (e.g., AND, OR, NOT) on its one or more binary inputs to create its single binary output.

Unlike conventional logic gates, the SOLGs can *both* receive signals from the traditional input *and* output terminals.
They are "terminal-agnostic". They accomplish this by *self-organizing* into their own logical proposition as well as in logical relations to another gate. For example, if a SOLG represents the logical operation OR, it has an internal mechanism that propels its terminal states to fulfill the relationship defined by $x_0 = x_1 \vee x_2$ (x_0 being the state of the traditional output terminal and x_1, x_2 being the states of the traditional input terminals). Then by setting x_0 will initiate the logic gate's self-organization to produce x_1 and x_2 states *consistent* with x_0 and the truth table of an OR gate. The ability to set the state of the output terminal is not available in traditional logic gates.

When SOLGs replace traditional logic gates, a self-organizing logic circuit (SOLC) is created.
The property of dynamic, *collective* self-organization of all the SOLGs in the circuit allows SOLCs to solve complex optimization problems from any state selected at random by evolving to converge into equilibrium points. These equilibria represent approximations that come closer to the global minimum than current best solutions.



A traditional Boolean circuit using logic Gate for prime factorization
It multiplies two integers p and q to give pq = n = 35 = $(100011)_2$ (in the little-endian notation).



Moving from traditional gates to terminal-agnostic gates

Nonlocal Collective State Computation

The most significant feature of SOLGs is its manifested *long-range order*. Long-range order describes physical systems which demonstrate correlated behavior across remote particles. In other words, systems with long-range order contain components that correspond to the states of other components regardless of distance. This simultaneous, collective responsiveness of individual parts describes the temporal and spatial non-locality of the system.

The capability of SOLGs to realize long-range order is due to the existence of instantons. Instantons connect topologically inequivalent critical points in the phase space. Instantons are the classical analogue of quantum tunneling. They create non-locality in the system which generate the collective, dynamic behavior of SOLGs to correlate at an arbitrary distance. In effect, this collective behavior allows SOLGs to efficiently adapt their truth value to satisfy the logical proposition of another gate without violating their own internal logical proposition. The nonlocality of SOLG allows for simultaneous variable flips which is the necessary but unaccomplishable task of combinatorial approaches once they reach a certain number of satisfied constraints.

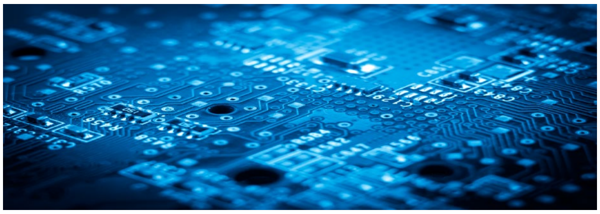
It is precisely the long-range order of SOLCs that allows for computation acceleration by orders of magnitude. As discussed in greater detail in the next section on the demonstrated performance of MemCPU Coprocessor, the system converges quickly to the equilibrium points which represent current closest approximations to the optimal solution to optimization problems.

Scalability in Linear Time

A key feature of the MemComputing solution is the ability of DMMs to scale in linear time. It is important to note that the linear scaling is independent of the input size because the number of logic gates grows linearly with each step requiring only a linear number of floating-point operations and linearly growing memory. In other words, the number of variables can be increased without an exponential growth in computation time which resolves the primary issue with conventional computation solutions.

The configuration of DMMs outlined here is able to s**upport infinite-range correlations in the infinite size limit**. This support allows for an ideal scale-free behavior of the SOLC in which the correlations do not decay. This was derived analytically using topological field theory.[2]

It is important to note that the linear scalability of DMMs does not prove the availability of polynomial solutions to NP-hard problems. Nevertheless, **the outstanding performance of a physics-based approach to solving complex optimization problems** demonstrates a promising trajectory for advancements in optimization computation.



3 Demonstrated Non-Combinatorial Approach to Optimization Problems

In order to demonstrate the extraordinary performance of MemComputing solutions, we compared MemCPU Coprocessor with the best solvers of the 2016 MaxSAT competition. An initial note is that while MemCPU Coprocessor is a single threaded interpreted MatLab application. The best solvers from the MaxSAT competition, CCLS and DeciLS are compiled software solutions. Interpreted MatLab is notoriously inefficient compared to compiled software. However, even though the comparison is not equivalent in every respect, the results captured the linear scaling of the MemComputing solver which offered better approximations in the least amount of time regardless of the increasing number of variables. Further, MemCPU Coprocessor scales linearly vs. exponentially. This indicates that the orders of magnitude performance improvement of MemCPU Coprocessor increases as the number of inputs increase.

The Challenge

Three types of problems were tested (in order of increasing difficulty): random-Max-E3SAT, hyper-Max-E3SAT, and delta-Max-E3SAT. Two contestants benchmarking MemCPU Coprocessor performance: CCLS and DeciLS (best solvers in the 2016 MaxSAT competition).

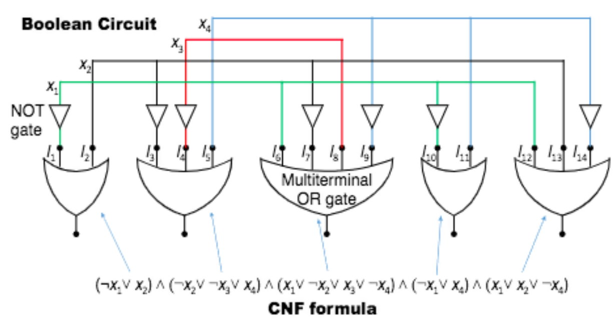
Given its NP-hardness, MaxSAT algorithms experience an inapproximability gap which says that no approximation can surpass a fraction of the globally optimal solution without incurring exponential overhead. Particularly for Max-E3SAT, the best approximation, under the conditions that NP ≠ P, is ⅞ of the optimal number of satisfied constraints.[3]

The Procedure

<u>Step 1</u>: Construct a Boolean circuit that represents the optimization problem

•All optimization problems can be written in Boolean format and any standard Boolean formula can be written in conjunctive normal form (CNF)

Boolean circuit representation of MaxSAT



<u>Step 2:</u> Replace traditional uni-directional Boolean gates with SOLGs
<u>Step 3:</u> Feed the appropriate terminals with the required output (for instance, the logical 1 if we are interested in checking its satisfiability)
<u>Step 4:</u> The circuit is represented by non-linear ordinary differential equations which can be solved numerically to find the equilibrium points. Given the collective behavior of the circuit, the equilibria come extremely close to the global optimum

The Result

With a threshold of 1.5% of unsatisfiable clauses, all three solvers were tested for the length of time it would take to surpass the limit with an increasing number of clauses for the hardest cases, namely those of the delta-Max-E3SAT.

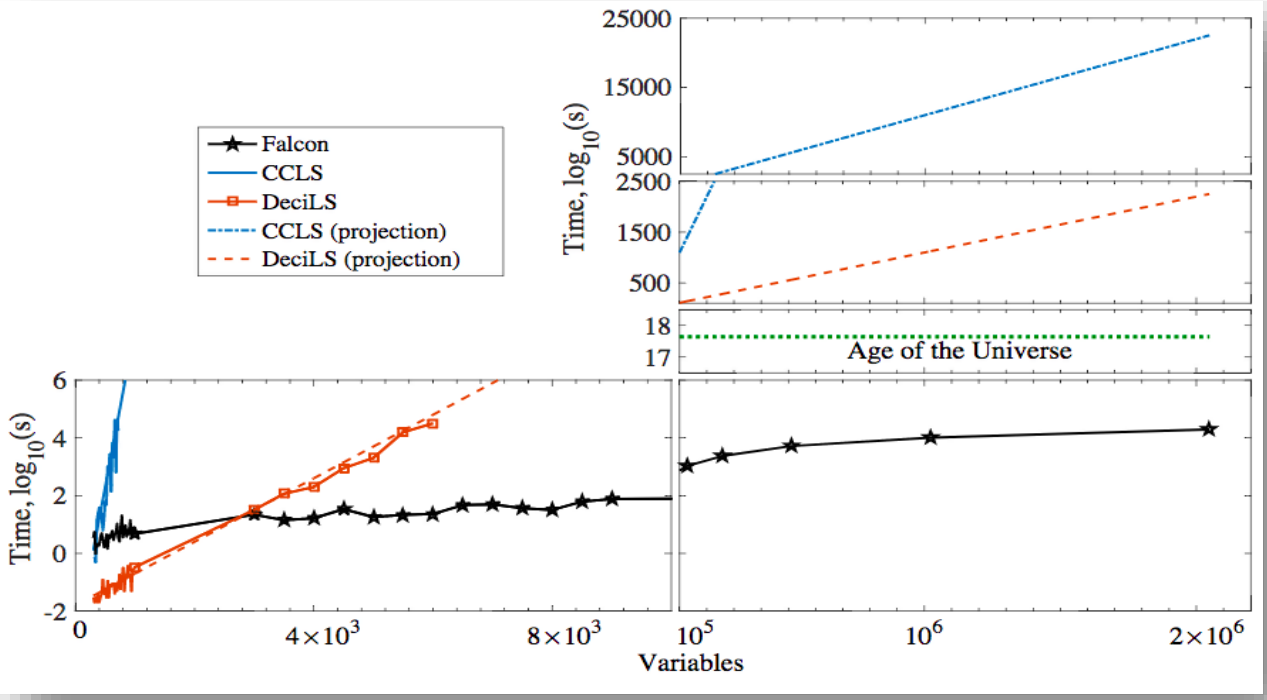


FIG. 1. Time comparison between CCLS, DeciLS, and Falcon for delta-Max-E3SAT problem with a set threshold of unsatisfiable clauses at 1.5%. The projected times for CCLS and DeciLS was estimated using a linear regression of the Log_{10}(time) versus the number of variables.

The exponential blowup of both CCLS and DeciLS is immediately apparent in Fig. 1. In comparison, Falcon's non-combinatorial approach scaled linearly for up to 2×10^6 variables which necessitated 10^4 seconds (a little more than a couple of hours) to reach the unsatisfiable clause limit at 1.5%. Had CCLS and DeciLS been required to compute 2×10^6 variables, the computation time would, at best, run for 10^{2500} seconds which is 10^{2480} times the estimated age of the universe.

4 Conclusion

The standard electronic circuitry utilized in MemComputing technology allows us to reproduce DMMs in software form. Our first product, MemCPU Coprocessor, is now available on the cloud in the form of a Software as a Service (SaaS) and through custom Software Development Kits (SDKs).

Currently, the MemCPU Coprocessor SaaS model offers a free evaluation period for users to submit problems in Conjunctive Normal Form, standard for Boolean propositions, to test the speed and quality of Falcon for themselves.

The next phase will involve the release of a series of software coprocessors in addition to hardware products such as an FPGA-based solution.

Ultimately, MemComputing aims at developing a MemCPU which is an integrated circuit that performs calculations in real time.

Reference

[1] Traversa, Fabio L., Pietro Cicotti, Forrest Sheldon, and Massimiliano Di Ventra. Evidence of an exponential speed-up in the solution of hard optimization problems. *arXiv preprint arXiv:1710.09278* (2017).

[2] Di Ventra, M., Traversa, F. L. & Ovchinnikov, I. V. *Topological field theory and computing with instantons*. Ann. Phys. (Berlin) (in press).

[3] Hastad, J. Some optimal inapproximability results. *Journal of the ACM* 48, 798-859 (2001).

MemComputing, Inc.'s disruptive coprocessor technology is accelerating the time to find feasible solutions to the most challenging operations research problems in all industries. Using physics principles, this novel software architecture is based on the logic and reasoning functions of the human brain.

MemComputing enables companies to analyze huge amounts of data and make informed decisions quickly, bringing efficiencies to areas of operations research such as Big Data analytics, scheduling of resources, routing of vehicles, network and cellular traffic, genetic assembly and sequencing, portfolio optimization, drug discovery and oil and gas exploration.

The company was formed by the inventors of MemComputing, PhD Physicists Massimiliano Di Ventra & Fabio Traversa and successful serial entrepreneur, John A. Beane.

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